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Quark Masses and Flat Directions in String Models

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Abstract

I discuss a method for producing a quasi–realistic inter–generational quark mass hierarchy in string models. This approach involves non–Abelian singlet states developing intermediate scale vacuum expectation values. I summarize recent investigations into string model realization of this.

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I. Quark Mass Hierarchy

The experimental measurement a few years ago of the top quark mass [1] eliminated the last unknown among the physical masses of *all* three generations of up–, down–, and electron–like particles. The masses of these particles are displayed in Table I, where I have expressed all masses in top–quark mass units. To first order, the inter–generational mass ratio for up–type quarks is $10^{-5} : 10^{-3} : 1$, and for down–type is $10^{-5} : 10^{-3} : 10^{-2}$.

In minimal supersymmetric standard model (MSSM) physics, quarks (and their supersymmetric partners) gain mass through superpotential couplings to Higgs bosons H_1 and H_2,*

$$W_{u_i} \sim \lambda_{u_i} \hat{H}_2 \hat{Q}_i \hat{U}^c_i \, ; \quad W_{d_i} \sim \lambda_{d_i} \hat{H}_1 \hat{Q}_i \hat{D}^c_i \, ; \tag{1}$$

where i is the generation number. Effective mass terms appear when the Higgs acquire a typical soft supersymmetry breaking scale vacuum expectation value (VEV) $\langle H_{1,2} \rangle \sim m_{soft} = \mathcal{O}(M_Z)$.†

Inter–generational mass ratios can be induced when the associated first and second generation superpotential terms contain effective couplings λ that include non–renormalizable suppression factors,

$$\lambda_{(u,d,e)_i} \sim (\frac{\langle S \rangle}{M_{\rm Pl}})^{P'_i}, \qquad \text{for } i = 1, 2, \tag{2}$$

where S is a non–Abelian singlet, $M_{\rm Pl}$ is the Planck scale (which is replaced by the $M_{\rm string}$ for string models), and P is a positive integer. VEVs only slightly below the Planck/string scale (often resulting from a $U(1)$ anomaly cancellation) imply large values of $P'_{1,2}$ for 10^{-5} and 10^{-3} suppression factors. In contrast, intermediate scale VEVs (between $M_{\rm Z}$ and $M_{\rm string}$) require far lower values for P'_i. In a series of recent papers intermediate scales have been explored [2,3] and their realization in actual models has been investigated [4,5]. This investigation has been the product of a fruitful collaboration with M. Cvetič, J. Espinosa, L. Everett, P. Langacker, & J. Wang at the University of Pennsylvania.‡ In the following section, I show how intermediate scales can occur and how, in theory, they could produce an inter–generational $10^{-5} : 10^{-3} : 1$ up–quark mass ratio, and a corresponding $10^{-5} : 10^{-3} : 10^{-2}$ down–quark mass ratio.

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* $\hat{X}$ denotes a generic superfield and X its bosonic component.

† Similarly, the three generations of electron–type leptons gain mass via superpotential terms $W_{e_i} \sim \lambda_{e_i} \hat{H}_1 \hat{L}_i \hat{E}^c_i$.

‡ In addition to this mass ratio study from the perturbative string theory perspective, Katsumi Tanaka of the Ohio State University and I have been investigating quark mass ratios from the non–perturbative approach of Seiberg–Witten duality [6].

II. Theoretical Quark Mass Hierarchy from String Models

One method by which an intermediate scale VEV $\langle S\rangle$ can generate non–renormaliz-able suppression factors involves extending the SM gauge group by an additional non–anomalous $U(1)'$. This approach requires (at least) two SM singlets S_1 and S_2, carrying respective $U(1)'$ charges Q'_1 and Q'_2. D–flatness for the non–anomalous $U(1)'$,

$$\langle D\rangle_{U(1)'} \equiv Q'_1|\langle S_1\rangle|^2 + Q'_2|\langle S_2\rangle|^2 = 0, \tag{3}$$

necessitates that Q'_1 and Q'_2 be of opposite sign. Together the VEVs of S_1 and S_2 form a D–flat scalar field direction S defined by,

$$\langle S_1\rangle = \cos\alpha_Q\langle S\rangle, \quad \langle S_2\rangle = \sin\alpha_Q\langle S\rangle, \quad \text{where} \quad \tan^2\alpha_Q \equiv \frac{|Q_1|}{|Q_2|}. \tag{4}$$

The F–flatness constraints

$$\langle F_{S_p}\rangle \equiv \langle\frac{\partial W}{\partial S_p}\rangle = 0,\ p = 1,2; \quad \text{and} \quad \langle W\rangle = 0, \tag{5}$$

imply that the D–flat direction $S = S_1\cos\alpha_Q + S_2\sin\alpha_Q$ is also a *renormalizable* F–flat direction if (as I assume hereon) $\hat{S}_1$ and $\hat{S}_2$ do not couple among themselves in the renormalizable superpotential.

Consider the real component of this flat direction, $s = \sqrt{2}\mathrm{Re}S = s_1\cos\alpha_Q + s_2\sin\alpha_Q$. This scalar's renormalization group equation (RGE) running mass is,

$$m^2 = m_1^2(\mu)\cos^2\alpha_Q + m_2^2(\mu)\sin^2\alpha_Q = \left(\frac{m_1^2}{|Q_1|} + \frac{m_2^2}{|Q_2|}\right)\frac{|Q_1Q_2|}{|Q_1| + |Q_2|}, \tag{6}$$

which generates a potential

$$V(s) = \frac{1}{2}m(\mu = s)^2 s^2. \tag{7}$$

I will assume that m^2 is positive at the string scale and of order $m^2_{\text{soft}} \sim \mathcal{O}(M_{\mathrm{Z}}{}^2)$ (m_o^2 if universality is assumed). However, through RGE running, m^2 can be driven negative (with electroweak (EW) scale magnitude) by large Yukawa couplings (i) of S_1 to exotic triplets, $W = h\hat{D}_1\hat{D}_2\hat{S}_1$; (ii) of S_1 to exotic doublets and of S_2 to exotic triplets, $W = h_D\hat{D}_1\hat{D}_2\hat{S}_1 + h_L\hat{L}_1\hat{L}_2\hat{S}_2$; or (iii) of S_1 to varying numbers of additional SM singlets $W = h\sum_{i=1}^{N_p}\hat{S}_{ai}\hat{S}_{bi}\hat{S}_1$ [2]. $m(\mu = s)^2$ can turn negative anywhere between a scale of $\mu_{rad} = 10^4$ GeV and $\mu_{rad} = 10^{17}$ GeV (slightly below the string scale) for various choices of the supersymmetry breaking parameters A^0 (the universal Planck scale soft trilinear coupling) and $M_{1/2}$ (the universal Planck scale gaugino mass).[§]

[§]The standard universal scalar EW soft mass–squared parameter m_0 has a simple normalizing effect, with A^0/m_0 and $M_{1/2}/m_0$ being the actual relevant parameters.

When m^2 runs negative, a minimum of the potential develops along the flat direction and S gains a non–zero VEV. In the case of only a mass term and no Yukawa contribution to $V(s)$, minimizing the potential

$$\frac{dV}{ds} = \left(m^2 + \frac{1}{2}\beta_{m^2} \right)\Big|_{\mu=s} s = 0, \tag{8}$$

(where $\beta_{m^2} = \mu \frac{dm^2}{d\mu}$) shows that the VEV $\langle s \rangle$ is determined by

$$m^2(\mu = \langle s \rangle) = -\frac{1}{2}\beta_{m^2}. \tag{9}$$

Eqs. (8,9) are satisfied very close to the scale μ_{RAD} at which m^2 crosses zero. μ_{RAD} is fixed by the renormalization group evolution of parameters from $M_{\rm string}$ down to the EW scale and will lie at some intermediate scale.

Location of the potential minimum can also be effected by non–renormalizable self–interaction terms,

$$W_{\rm NR} = \left(\frac{\alpha_K}{M_{\rm Pl}} \right)^K \hat{S}^{3+K}, \tag{10}$$

where $K = 1, 2...$ and α_K are coefficients. Such non–renormalizable operators (NRO's) lift the flat direction (by breaking F–flatness) for sufficiently large values of s.

The general form of the potential, $V(X_p)$, for the scalar components X_p of corresponding supermultiplets $\hat{X}_p$ is

$$V(X_p) = V_{soft\ susy} + \sum_p \mid \frac{\partial W}{\partial \hat{X}_p} \mid^2 + \frac{1}{2} g_\alpha^2 \sum_\alpha \mid \sum_p Q_p^\alpha |X_p\mid^2|^2 \tag{11}$$

$$= V_{soft\ susy} + \sum_p \mid F_p \mid^2 \quad + \frac{1}{2} g_\alpha^2 \sum_\alpha \mid D_\alpha \mid^2 \tag{12}$$

Thus, NRO contributions transform $V(s)$ in eq. (7) into

$$V(s) = \frac{1}{2} m^2 s^2 + \frac{1}{2(K+2)} \left(\frac{s^{2+K}}{\mathcal{M}^K} \right)^2, \tag{13}$$

where $\mathcal{M} = \mathcal{C}_K M_{\rm Pl}/\alpha_K$, with $\mathcal{C}_K = [2^{K+1}/((K+2)(K+3)^2)]^{1/(2K)}$.

Even when an NRO is present, the running mass effect still dominates in determining $\langle s \rangle$ if $\mu_{RAD} \ll 10^{12}$ GeV. However, an NRO is the controlling factor when $\mu_{RAD} \gg 10^{12}$ GeV. In the latter case, we find that

$$\langle s \rangle = \left[\sqrt{(-m^2)} \mathcal{M}^K \right]^{\frac{1}{K+1}} = \mu_K \sim (m_{soft} \mathcal{M}^K)^{\frac{1}{K+1}}, \tag{14}$$

where $m_{soft} = \mathcal{O}(|m|) = \mathcal{O}(M_Z)$ is a typical soft supersymmetry breaking scale. While $\langle s \rangle$ is an intermediate scale VEV, the mass M_S of the physical field s is still on the order of the soft SUSY breaking scale: For running mass domination,

$$M_S^2 \equiv \frac{d^2V}{ds^2}\bigg|_{s=\langle s\rangle} = \left(\beta_{m^2} + \frac{1}{2}\mu\frac{d}{d\mu}\beta_{m^2}\right)\bigg|_{\mu=\langle s\rangle} \simeq \beta_{m^2} \sim \frac{m_{soft}^2}{16\pi^2}, \tag{15}$$

while in the NRO–controlled case,

$$M_S^2 = 2(K+1)(-m^2) \sim m_{soft}^2. \tag{16}$$

What powers P'_i in (2) for first and second generation suppression factors in an NRO–dominated model could produce an up–type quark mass ratio of order $10^{-5} : 10^{-3} : 1$? From eq. (14), we see the suppression factors become

$$\left(\frac{m_{soft}}{M}\right)^{\frac{P'_i}{K+1}}, \tag{17}$$

where the coefficient α_K has been absorbed into the definition of the mass scale $M \equiv \mathcal{M}/\mathcal{C}_K$. The mass suppression factors for specific P' (in the range 0 to 5) and K (in the range of 1 to 7) are given in Table II. From this table we find that the choices $P'_1 = 2$ and $P'_2 = 1$ in tandem with $K = 5$ or $K = 6$ (for the self–interaction terms of S) can, indeed, reproduce the required mass ratio. These values of K invoke an intermediate scale $\langle S\rangle$ around 8×10^{14} GeV to 2×10^{15} GeV,

The first and second generation down–quark suppression factors can be similarly realized. However, unless $tan\ \beta \equiv \frac{\langle H_2\rangle}{\langle H_1\rangle} \gg 1$, the intra–generational mass ratio of $10^{-2} : 10^{-2} : 1$ for m_τ, m_b, and m_t is not realizable from a $K = 5$ or 6 NRO singlet term. For $tan\ \beta \sim 1$, m_τ, m_b are too small to be associated with a renormalizable coupling ($P = 0$) like that assumed for m_t, but are somewhat larger than predicted by $P = 1$ for $K = 5$ or $K = 6$. Instead, m_b and m_τ might be associated with a different NRO involving the VEV of an entirely different singlet. In that event, Table II suggests another flat direction S' (formed from a second singlet pair S'_1 and S'_2), with a $K = 7$ self–interaction NRO and $P'_3 = 1$ suppression factor for m_b and m_τ.**

III. Realization of Quark Mass Hierarchy in String Models

In string models, one problem generally appears at the string scale that must be resolved "before" possible intermediate scale flat direction VEVs can be investigated. That is, most four–dimensional quasi–realistic $SU(3)_C \times SU(2)_L \times U(1)_Y$ string models contain an anomalous $U(1)_A$ (meaning $\mathrm{Tr}Q_A \neq 0$) [7]. In fact, in a generic charge basis, a string model with an Abelian anomaly may actually contain not just one, but several anomalous $U(1)$ symmetries. However, all anomalies can all be transferred into a single $U(1)_A$ through the *unique* rotation

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**An intermediate scale VEV $\langle S\rangle$ can also solve the μ problem through a superpotential term $W_\mu \sim \hat{H}_1\hat{H}_2\hat{S}\left(\frac{\hat{S}}{M}\right)^{P_\mu}$. With NRO–dominated $\langle S\rangle \sim (m_{soft}M^K)^{\frac{1}{K+1}}$, the effective Higgs μ–term takes the form, $\mu_{eff} \sim m_{soft}\left(\frac{m_{soft}}{M}\right)^{\frac{P-K}{K+1}}$. The phenomenologically preferred choice among this class of terms is clearly $P = K$: this yields a K–independent $\mu_{eff} \sim m_{soft}$.

$$U(1)_{\rm A} \equiv c_A \sum_n \{{\rm Tr} Q_n\} U(1)_n, \tag{18}$$

with c_A a normalization factor. The remaining non–anomalous components of the original set of $\{U(1)_n\}$ may be rotated into a complete orthogonal basis $\{U(1)_a\}$.

The standard anomaly cancellation mechanism [8,9] breaks $U(1)_{\rm A}$ at the string scale, while simultaneously generating a FI D–term,

$$\xi \equiv \frac{g_s^2 M_P^2}{192\pi^2} {\rm Tr} Q_A \, , \tag{19}$$

where g_s is the string coupling and M_P is the reduced Planck mass, $M_P \equiv M_{Planck}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$. The FI D–term breaks spacetime supersymmetry unless it is cancelled by appropriate VEVs $\langle X_p \rangle$ of scalars X_p that carry non–zero anomalous charge,

$$\langle D \rangle_{\rm A} \equiv \sum_j Q_j^{(A)} |\langle X_p \rangle|^2 + \xi = 0 \ . \tag{20}$$

Generalizations of D– and F–constraints for $S_{1,2}$ (i.e., of eqs. (3) and (5)) are imposed on possible VEV directions $\{\langle X_p \rangle\}$: $\langle D \rangle_a \equiv \sum_p Q_p^{(a)} |\langle X_p \rangle|^2 = 0$ and $\langle F_p \rangle \equiv \langle \frac{\partial W}{\partial \hat{X}_p} \rangle = 0$; $\langle W \rangle = 0$.

My colleagues and I at Penn have developed methods for systematically determining [10,11] and classifying [3] D– and F–flat directions in string models. We have applied this process [4,5] to the free fermionic three generation $SU(3)_C \times SU(2)_L \times U(1)_Y$ models (all of which contain an anomalous $U(1)_A$) introduced in refs. [12], [13] and [14]. For each model, we have determined the anomaly cancelling flat directions that preserve hypercharge, only involve VEVs of non–Abelian singlet fields, and are F–flat to all orders in the non–renormalizable superpotential.

Flat directions in Model 5 of [14] have particularly received our attention [4]. In Model 5 we investigated the physics implications of various non–Abelian singlet flat directions. After breaking the anomalous $U(1)_A$, all of these flat directions left one or more additional $U(1)_a$ unbroken at the string scale. For each flat direction, the complete set of effective mass terms and effective trilinear superpotential terms in the observable sector were computed *to all orders* in the VEV's of the fields in the flat direction. The "string selection–rules" disallowed a large number of couplings otherwise allowed by gauge invariance, resulting in a massless spectrum with a large number of exotics,[††] which in most cases are excluded by experiment. This signified a generic flaw of these models. Nevertheless, we found the resulting trilinear couplings of the massless spectrum to possess a number of interesting features which we analyzed for two representative flat directions. We investigated the fermion texture; baryon– and lepton–number violating couplings; R–parity breaking; non–canonical μ terms; and

[††]Recently it was shown [15] that free fermionic construction can actually provide string models wherein *all* MSSM exotic states gain near–string–scale masses via flat direction VEVs, leaving only a string–generated MSSM in the observable sector below the string scale. The model of ref. [12] was presented as the first known with these properties.

the possibility of electroweak and intermediate scale symmetry breaking scenarios for a $U(1)' \in \{U(1)_a\}$. The gauge coupling predictions were obtained in the electroweak scale case. We found $t-b$ and $\tau-\mu$ fermion mass universality, with the string scale Yukawa couplings g and $g/\sqrt{2}$, respectively. Fermion textures existed for certain flat directions, *but only in the down–quark sector.* Lastly, we found baryon– and lepton– number violating couplings that could trigger proton–decay, $N-\bar{N}$ oscillations, leptoquark interactions and R–parity violation, leading to the absence of a stable LSP.

IV. Comments

I have discussed how intermediate VEVs hold the potential to yield quasi–realistic quark mass ratios. Four–dimensional string models usually require cancellation of the FI D–term contribution from an anomalous $U(1)_A$ by near string scale VEVs. Since some non–anomalous $U(1)_a$ are simultaneously broken at the string scale by these VEVs, which $U(1)_a$ might be associated with intermediate scale VEVs strongly depends on the particular set of (near) string–scale VEVs chosen. As our investigations into flat directions have demonstrated, various choices for flat VEV directions can drastically alter low energy phenomenology. The textures of the quark mass matrices can be strongly effected by choice of flat direction since textures are wrought by effective mass terms in the non–renormalizable superpotential.

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m_u	:	m_c	:	m_t	=	3×10^{-5}	:	7×10^{-3}	:	1
m_d	:	m_s	:	m_b	=	6×10^{-5}	:	1×10^{-3}	:	3×10^{-2}
m_e	:	m_μ	:	m_τ	=	0.3×10^{-5}	:	0.6×10^{-3}	:	1×10^{-2}

TABLE I. Fermion mass ratios with the top quark mass normalized to 1. The values of $u-$, $d-$, and s-quark masses used in the ratios (with the t-quark mass normalized to 1 from an assumed mass of 170 GeV) are estimates of the $\overline{\text{MS}}$ scheme current-quark masses at a scale $\mu \approx 1$ GeV. The c- and b-quark masses are pole masses.

	$P^{(\prime)}$	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	$K=7$
$\left(\frac{m_{soft}}{M}\right)^{\frac{1}{K+1}}$		2×10^{-8}	7×10^{-6}	1×10^{-4}	8×10^{-4}	3×10^{-3}	6×10^{-3}	1×10^{-2}
$\langle S \rangle$ (GeV)		5×10^{9}	2×10^{12}	4×10^{13}	2×10^{14}	8×10^{14}	2×10^{15}	3×10^{15}
$\frac{\mu_{eff}}{m_{soft}}$	$K-1$	5×10^{7}	1×10^{5}	7×10^{3}	1×10^{3}	400	200	90
	K	1	1	1	1	1	1	1
	$K+1$	2×10^{-8}	7×10^{-6}	1×10^{-4}	8×10^{-4}	3×10^{-3}	6×10^{-3}	1×10^{-2}
$\frac{m_{Q,L}}{\langle H_i \rangle}$	0	1	1	1	1	1	1	1
	1	2×10^{-8}	7×10^{-6}	1×10^{-4}	8×10^{-4}	3×10^{-3}	6×10^{-3}	1×10^{-2}
	2	3×10^{-16}	5×10^{-11}	2×10^{-8}	6×10^{-7}	7×10^{-6}	4×10^{-5}	1×10^{-4}
	3	6×10^{-24}	3×10^{-16}	2×10^{-12}	5×10^{-10}	2×10^{-8}	2×10^{-7}	2×10^{-6}
	4	1×10^{-31}	2×10^{-21}	3×10^{-16}	4×10^{-13}	5×10^{-11}	1×10^{-9}	2×10^{-8}
	5	2×10^{-39}	2×10^{-26}	5×10^{-20}	3×10^{-16}	1×10^{-13}	9×10^{-12}	2×10^{-10}

TABLE II. Non-Renormalizable MSSM mass terms via $\langle S \rangle$. For $m_{soft} \sim 100$ GeV, $M \sim 3 \times 10^{17}$ GeV.

REFERENCES

[1] R.M. Barnett *et al.*, *Phys. Rev.* **D54** (1996) 1.

[2] G. Cleaver, M. Cvetič, L. Everett, J.R. Espinosa, and P. Langacker, *Phys. Rev.* **D57** (1997) 2701, [hep-ph/9705391].

[3] G. Cleaver, M. Cvetič, L. Everett, J.R. Espinosa, and P. Langacker, *Nucl. Phys.* **B525** (1998) 3, [hep-th/9711178]; "Flat Directions in Three Generation Free Fermionic String Models," UPR-0784-T, [hep-th/9805133].

[4] G. Cleaver, M. Cvetič, L. Everett, J.R. Espinosa, P. Langacker, and J. Wang, "Physics Implications of Flat Directions in Free Fermionic Superstring Models I: Mass Spectrum and Couplings," CERN-TH/98-243, UPR-0811-T, IEM-FT-177/98, UM-TH-98/17 [hep-th/9807479]; "Physics Implications of Flat Directions in Free Fermionic Superstring Models II: Renormalization Group Analysis," UPR-0814-T, UM-TH-98/18, [hep-th/9811355]

[5] G. Cleaver, "Phenomenological Aspects of CHL Three Generation String Models," UPR-0772-T, to appear.

[6] G. Cleaver and K. Tanaka, "Ratio of Quark Masses in Duality Theories," work in progress.

[7] For general discussions of anomalous $U(1)$ in string models and further lists of references see, e.g.,
T. Kobayashi and H. Nakano, *Nucl. Phys.* **B496** (97) 103, [hep-th/9612066];
G. Cleaver, *Nucl. Phys.* **B** (Proc. Suppl.) **62A-C** (1998) 161 [hep-th/9708023];
G. Cleaver and A.E. Faraggi, UFIFT-HEP-97-28, UPR-0773-T, [hep-ph/9711339];
A.E. Faraggi, *Phys. Lett.* **B426** (98) 315, [hep-ph/9801409];
L.E. Ibáñez, R. Rabadan, and A.M. Uranga, [hep-th/9808139];
P. Ramond, Proceedings of Orbis Scientiae '97 II, Dec. 1997, Miami Beach, [hep-ph/9808488];
W. Pokorski and G.G. Ross, [hep-ph/9809537].

[8] M. Dine, N. Seiberg, and E. Witten, *Nucl. Phys.* **B289** (1987) 585.

[9] J. Atick, L. Dixon, and A. Sen, *Nucl. Phys.* **B292** (87) 109;
M. Dine, I. Ichinose, and N. Seiberg, *Nucl. Phys.* **B293** (87) 253;
M. Dine and C. Lee, *Nucl. Phys.* **B336** (90) 317;
L. Dixon and V. Kaplunovsky, unpublished.

[10] See also:
M. Luty, *Phys. Rev.* **D53** (96) 3399, [hep-th/9506098];
T. Gherghetta, C. Kolda, and S. Martin, *Nucl. Phys.* **B468** (96) 37, [hep-th/9510370];
E. Dudas, C. Grojean, S. Pokorski, and C.A. Savoy, *Nucl. Phys.* **B481** (96) 85, [hep-ph/9606383];
P. Binetruy, N. Irges, S. Lavignac, P. Ramond, *Phys. Lett.* **B403** (97) 38, [hep-ph/9612442];
N. Irges and S. Lavignac *Phys. Lett.* **B424** (98) 293, [hep-ph/9712239].

[11] G. Cleaver, "Mass Hierarchy and Flat Directions in String Models," UPR-0795T, Proceedings of Orbis Scientiae '97 II, Dec. 1997, Miami Beach.

[12] A. Faraggi, D.V. Nanopoulos, and K. Yuan, *Nucl. Phys.* **B335** (1990) 347.

[13] A. Faraggi, *Phys. Lett.* **B278** (1992) 131; *Phys. Lett.* **B274** (1992) 47.

[14] S. Chaudhuri, G. Hockney, and J. Lykken, *Nucl. Phys.* **B469** (1996) 357.
[15] G. Cleaver, A. Faraggi, and D.V. Nanopoulos, "String Derived MSSM and M–Theory Unification," ACT-11/98, CTP-TAMU-45/98, TPI-MINN-98/24, UMN-TH-1729-98, [hep-ph/9811427].